MODELLING TRAFFIC FLOW ON MOTORWAYS: A HYBRID MACROSCOPIC APPROACH

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Abstract

The necessity for transportation research has been prompted by the exponential rise in vehicle use, research in the field has expanded and diversified accordingly. Despite substantial work having been carried out in Macroscopic modelling, the theory is far from complete [1]. The paper presents a hybrid model which combines the strengths of macroscopic models; the continuum method with forecasting models. The comparison presented herein is implemented using three different traffic flow forecasting methods: the artificial neural network, random walk and SARIMA. Each method is coupled with the Lighthill-Whitham and Richards continuum model [2,3]. The model is designed to function with speed and relative accuracy to allow for real-time applications. The results from numerical experiments using a case study (a junction in Hampshire, England) highlights the positive features of such a model in describing traffic dynamics.

1. Introduction

Within the scope of traffic flow modelling, macroscopic models have the highest level of aggregation and the lowest level of detail on a characteristic variable level. Variables of interests at this scale are average variables such as flow and density. Two common types of models are used: continuum models derived from fluid dynamic equations [2,3], and short term traffic flow forecasting models based on historical data. Continuum models are considered well suited to describing road traffic dynamics on continuous stretches but unsuitable to model traffic through junctions. Short-term traffic flow forecasting methods can forecast successfully traffic flow rates around intersections, utilising historical data collected locally, however they are rather crude when attempting to give a full description of traffic dynamics along road sections.

An accurate traffic flow model must be capable of handling the nonlinear dynamics along road sections as well as the complex flow patterns around intersections. This study will introduce a hybrid model combining the strengths of the two types of macroscopic modelling. The short-term forecasting component will be used to capture traffic patterns at intersections and traffic dynamics along road sections will be described using a continuum model. The hybrid algorithm presented has been implemented using three different traffic flow forecasting methods – artificial neural networks, random walk and SARIMA – each coupled with the Lighthill-Whitham and Richards continuum model [2,3]. Results from a case study, using data collected at a Hampshire (England) junction, confirm the promising features of the introduced approach.

2. Macroscopic Traffic Flow Modelling

2.1. Continuum Models

For this family of models, road traffic flow is viewed as an inviscid compressible fluid in a channel following the Burgers [4] hyperbolic conservation law. Such an approach describing road traffic flow has been introduced by Lighthill and Whitham, and separately Richards [2,3]. The developed model is known as the LWR model.

2.1.1. Lighthill-Whitham and Richards (LWR) model

Let $\rho(x,t)$ denote the average density of vehicles at position x at time t. The LWR model is given by:

$$\frac{\partial \rho}{\partial t} + \frac{\partial f(\rho)}{\partial x} = 0,$$

where $f(\rho) = \rho V(\rho)$, and the function $V(\rho)$ denotes the equilibrium velocity. The above equation is known as the fundamental law of continuum traffic theory and it describes the conservation of the number of vehicles along the road. The speed density relationship is considered as a decreasing function defined on the interval $[0, \rho_{\{max\}}]$where $\rho_{\{max\}}$is the maximum density possible for the given road, when normalized is equal to 1. Empirically the equilibrium velocity is given by

$$V(\rho) = V_{\{max\}} \left(1 - \frac{\rho}{\rho_{\{max\}}}\right)$$

where $V_{\{max\}}$ denotes the maximum allowed on an empty road therefore $V(0) = V_{\{max\}}$and $V_{\rho_{\{max\}}} = 0$. Thus the flow function is a concave function with a maximum σ given by $0 < \sigma < \rho_{\{max\}}$.

Lebaque [5] outlined the demand function $\rho \to d(\rho)$ given by the extension of the non-decreasing part of the function $f(\rho)$, and the supply function $\rho \to s(\rho)$ is given by the extension of the non increasing part of the function $f(\rho)$.
Hence,

$$d(\rho) = \begin{cases} f(\rho), & \rho \leq \sigma, \\ f(\sigma), & \rho > \sigma, \end{cases} \quad \text{and} \quad s(\rho) = \begin{cases} f(\rho), & \rho \geq \sigma, \\ f(\sigma), & \rho < \sigma. \end{cases}$$

Using the demand and supply functions, the LWR model can be solved numerically giving:

$$\rho_j^{n+1} = \rho_j^n + \frac{\Delta t}{\Delta x}\left(\Phi_{j-1}^n - \Phi_j^n\right)$$

where $\Phi_k^n = \min(demand(\rho_k^n), supply(\rho_{k+1}^n))$, when constructing the system. Initial road conditions must be provided i.e. the initial density state of the observed road at $t = 0$.

2.2. Short-term Traffic Flow Forecasting Models

Short-term traffic flow forecasting is used to predict traffic flow patterns in the short to immediate future. For the sake of the hybrid model, regression techniques and nodal modelling methods will be outlined.

2.2.1. Regression Techniques

The regression family of models and more importantly auto-regressive integrated moving average models (ARIMA) are well established within traffic flow research. Due to their simplicity and adaptability they provide a large toolbox with which to forecast traffic flow.

$ARIMA_{(p,d,q)}$model is defined by:

$$\phi(B)(1-B)^d y_t = \theta(B) Z_t$$

where y_t is the output for time t, Z_t is a white noise sequence. d is the level of differencing, B is the 'backshift operator' defined as $B^n y_t = y_{t-n}$. $\phi(B)$ is a polynomial of degree p (the number of autoregressive terms) and $\theta(B)$ is a polynomial of degree q (the number of lagged forecast errors in the prediction equation) given as:

$$\phi(B) = (1 - \alpha_1 B - \alpha_2 B^2 - \cdots - \alpha_p B^p)$$

$$\theta(B) = (1 - \beta_1 B - \beta_2 B^2 - \cdots - \beta_q B^q)$$

The α and β terms are calculated using historical data.

In this study we are also interested in the random walk and SARIMA models, two variants of the ARIMA family.

The random walk is often used as a baseline to judge the accuracy of other models due to its simplicity and forecasting speed (computationally). The random walk is defined by an $ARIMA_{(0,1,0)}$ model as follows:

$$y_t = y_{t-1} + Z_t$$

In traffic flow, the intrinsic seasonality of the data is due to the variations from day to day and the variation of the rush-hour. These variations are of particular interest so a Seasonal ARIMA model would seem more appropriate. The time series data containing a seasonal periodic component can be fitted to a SARIMA model [6]. The general multiplicative $SARIMA_{(p,d,q)(P,D,Q)_S}$ is as follows:

$$\phi(B)\Phi(B^S)(1-B)^d(1-B^S)^D y_t = \theta(B)\Theta(B^S)Z_t$$

where Φ, Θ, P, D, Q are the seasonal counterparts of ϕ, θ, p, d, q, respectively; and S denotes the seasonality. The value of S can be chosen by inspection, it is the value for which a noticeable trend in the data repeats e.g. for traffic flow, data collected in 15 min time intervals S=96, that is equal to one day of data.

2.2.2.Neural Networks

Neural networks are knowledge based tools used to identify complex relationships between inputs and outputs and/or to find patterns in the data. In terms of traffic data, the number of inputs and outputs is clearly defined by the number of entering and number of exiting roads at any junction. In this study we use the feed-forward structure of a neural network, it is most sufficient when compared to realistic traffic flow through a junction. It consists of neurons, simple processing units and directed, weighted connections between neurons. The weight of a connection between two neurons i and j is given by $w_{i,j}$. Using the hyperbolic tangent function as an activation function as suggested in the research [7], the values of the hidden vector h are given by:

$$h_j = \tanh(w_{i,j} q_i)$$

where q is the input vector. The output vector o is then defined by a weighted sum:

$$o_m = \sum_{j=1}^{p} w_{(j,m)} h_j$$

The value of i and m is given, in the traffic sense, by the number of inflow and outflow roads at an intersection. With a learning rate $a = [0,1]$ the weight update formulae is:

$$w_{j,m} = w_{j,m} - \varepsilon a \tanh(q_i w_{i,j})$$

$$w_{i,j} = w_{i,j} - \varepsilon a w_{j,m} \left(1 - \left(\tanh\left(q_i w_{i,j}\right)\right)^2\right) q_i$$

The architecture of the model allows, after suitable training, a much larger forecast horizon to be utilised in comparison to the standard regression formulae.

3. Hybrid Macroscopic Traffic Flow Modelling

In more recent times the short comings of macroscopic short-term forecasting models and continuum models has been realised; the response to which has been the development of hybrid macroscopic models, which aim to forecast the traffic dynamics of complex road networks with a high level of aggregation allowing for a faster computing time and therefore real time application in intelligent transportation systems [8,9]. This section proceeds in outlining a hybrid model combining the strengths of continuum based and short-term forecasting models.

The algorithm of the new hybrid approach performs as follows:

Step 1: Prescribe the initial conditions and boundary conditions for each road;
Step 2: Simulate traffic dynamics along stretches of road using a LWR model;

Step 3: Using current traffic flow-rates downstream and upstream the junction as inputs for the traffic flow forecasting model, forecast the traffic flow-rate passing through the junction and check the forecasted value using the traffic demand upstream the junction and the traffic supply downstream of the junction;
Step 4: Given traffic conditions in Step 2 and the flow-rate passing through the junction estimated in Step 3, determine the corresponding density on each road;
Step 5: Use the density values obtained in Step 4 as boundary conditions for the LWR model and then go to Step 2. Continue until time $= n$ (where n =forecast horizon)

The training of the weights for the neural network would be done at Step 1.

4. Case Study

To validate the proposed model, real sampled data is taken from 6 detectors placed at a motorway junction in Hampshire, England. Figure 1 shows the motorway junction. The real data from the collection points is used to inform the weights for the neural network and the historical data required for the regression models.

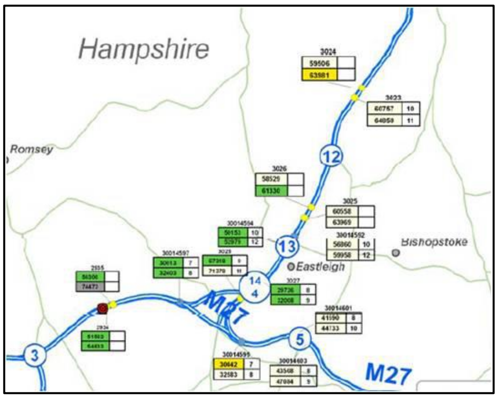


Figure 1: Case study road network (Hampshire, England)

The hybrid model is used to forecast three days of traffic through the junction, of the M27 feeding into the M3, the Mean Average Percentage Error (MAPE) and Normalised Root Mean Square Deviation (NRMSD) for each model is recorded as a measure of suitability and accuracy. The parameter values for the LWR model are: $v_{\{max\}} = 90\ km/h$ and $\rho_{\{max\}} = 140\ vehicles/km$. For the sake of simplicity dimensionless normalised density and velocity is considered. Hence the normalised maximal density and velocity are $\rho_{\{max\}} = v_{\{max\}}$. Each road is discretised into $m = 20$ sections and a free-flow boundary condition is set ($\rho = 0.5$ corresponding to σ in **Error! Reference source not found.**). The initial condition of the road resembles a sine wave to simulate natural traffic grouping at $t = 0$.

5. Results and Discussion

5.1. LWR – Random walk model

The average diagnostic results (over 500 iterations) from the hybridization are shown in Table 1. The LWR forecast for the combination can be seen in Figure 2, the layout of the graphs reflect the layout of the junction.

Table 1: LWR - Random Walk model diagnostic results

Model	MAPE(%)	NRMSD	Time (seconds)
LWR – Random Walk	3.4999	0.4975	0.1623

The results are as expected, the random walk is known to be fast, with a computation time to forecast three days ~ 0.16 seconds. Using an iterative updating scheme as is used in this hybridization provides a highly accurate forecast with a MAPE value of $\sim 3.5\%$. The random walk is often considered a baseline to compare statistical forecasting methods, therefore it shows the robustness of the algorithm in general when the diagnostic results are so encouraging.

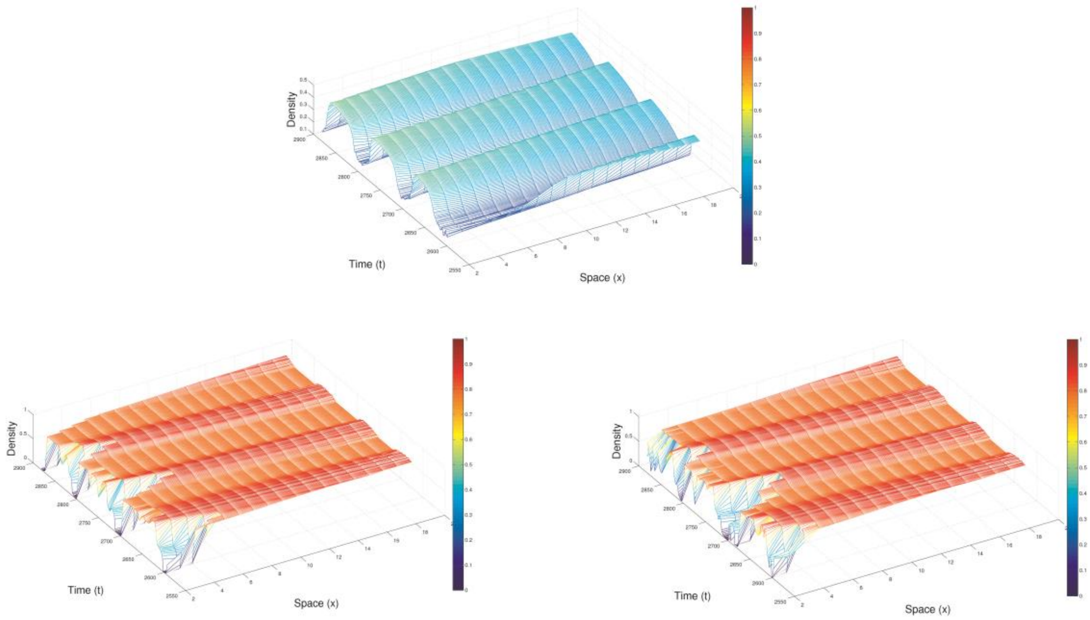


Figure 2: Hybrid LWR - Random Walk density forecast on the case study roads

The LWR forecast in Figure 2 shows the creation and dissipation of the characteristic kinematic waves. Since the exiting density of the downstream road (the road after the junction) is set to 0.5, the road is effectively capped at 0.5 so when the two roads have high densities entering the junction they are unable to all exit hence the congestion is generated. When both merging roads have low densities entering the network, the congestion dissipates until the next wave of traffic catches up, indicated in the figure by the blue areas.

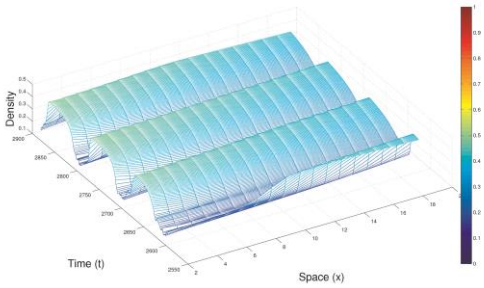
5.2. LWR – SARIMA model

The LWR – SARIMA model provides a fairly stable forecast at the junction, with MAPE, NRMSD and computation times comparable to the LWR – random walk model. The diagnostics for this combination are shown in Table 2. The LWR forecast for the combination can be seen in Figure 3. The forecast that it provides using an iterative updating method is robust and fast.

Table 2: LWR - SARIMA model diagnostic results

Model	MAPE (%)	NRMSD	Time (seconds)
LWR - SARIMA	4.1231	0.4846	0.1056

Regression techniques are considered to have a quick forecast time, computationally speaking. The SARIMA forecast diagnostics in Table 2 show this with a computation time of ~ 0.1 seconds. Using an iterative updating scheme the model performs accurately with a MAPE value of $\sim 4.1\%$.



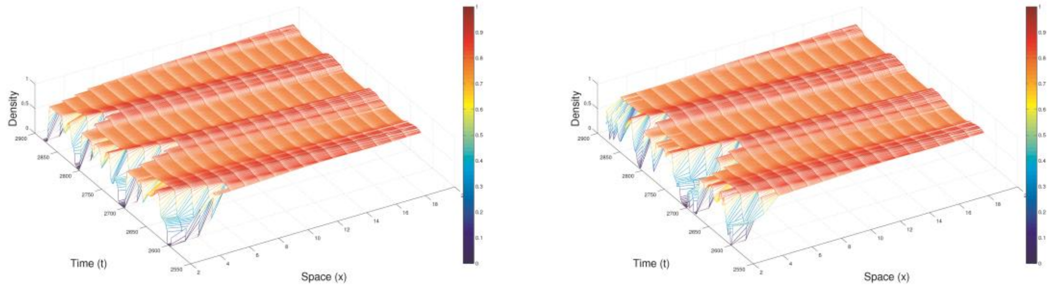


Figure 3: Hybrid LWR - SARIMA density forecast on the case study roads

The LWR forecasts in Figure 3 replicate the shock waves associated with traffic dynamics, indicating an accurate procedure. As with the LWR – random walk model the exiting density of the downstream road is set to 0.5, so the same bottleneck situation occurs.

5.3. LWR – NN model

The LWR – Neural Network model performs at the highest level of accuracy but also with the longest computation time. The diagnostics for this model are shown in Table 3. The LWR forecasts for this hybridization are shown in Figure 4.

Table 3: LWR - Neural Network diagnostic results

Model	MAPE(%)	NRMSD	Time (seconds)
LWR - NN	1.0477	0.0411	63.2103

The diagnostics table indicates that the model performs to a very high degree of accuracy with a MAPE value of $\sim 1\%$ but with a computation time of ~ 63 seconds. The performance of such a model is well documented and expected. Due to the nature of the learning process, the model can perform at a high level of accuracy for a longer amount of time unsupervised, but during the learning phase has the longest computation time. This result is in line with current research.

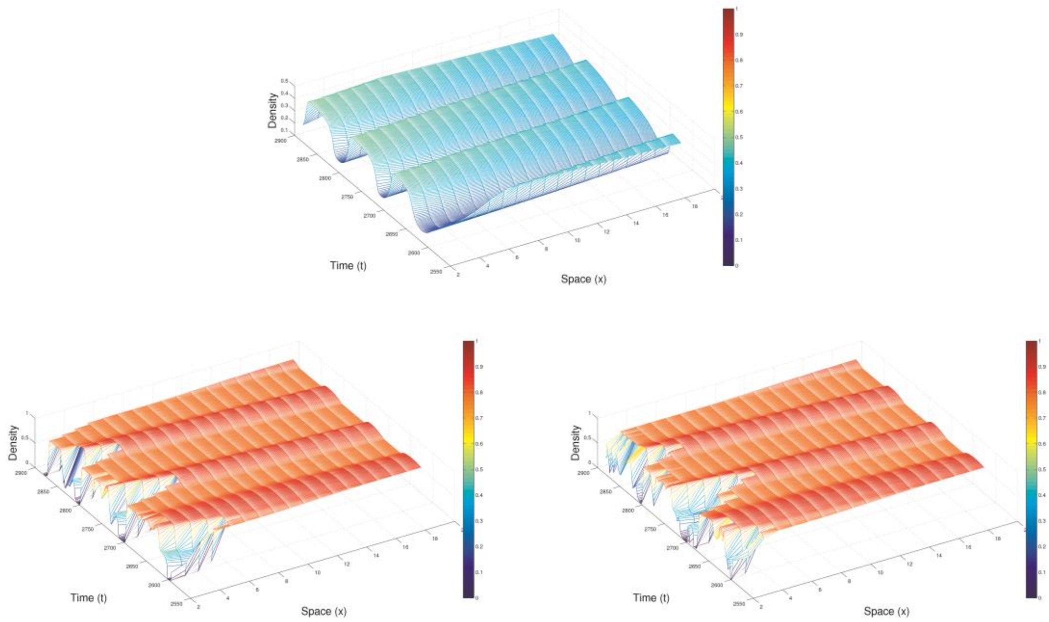


Figure 4: Hybrid LWR - NN density forecast on the case study roads

Due to the accuracy of the forecast at the junction, the LWR forecasts that were produced (Figure 4) show the greatest level of fidelity. The smooth traffic density peaks and troughs is evidence of the forecast. As with the other two models, the LWR forecasts show the creation and dissipation of the kinematic waves with the same bottleneck situation occurring.

5.4. Comparative discussion

Figure 5 shows the forecast for 3 days using the three methods updated iteratively. All three methods fit the data relatively well but the random walk and SARIMA show some quite large deviations from the observed data. The deviations are explained by the randomness of the stochastic Gaussian noise generator. Table 1 displays the diagnostic results of all three models combined.

Table 4: Model diagnostic results

Model	MAPE(%)	NRMSD	Time (seconds)
LWR – Random Walk	3.4999	0.4975	0.1623
LWR – SARIMA	4.1231	0.4846	0.1056
LWR – Neural Network	1.0447	0.0411	63.2103

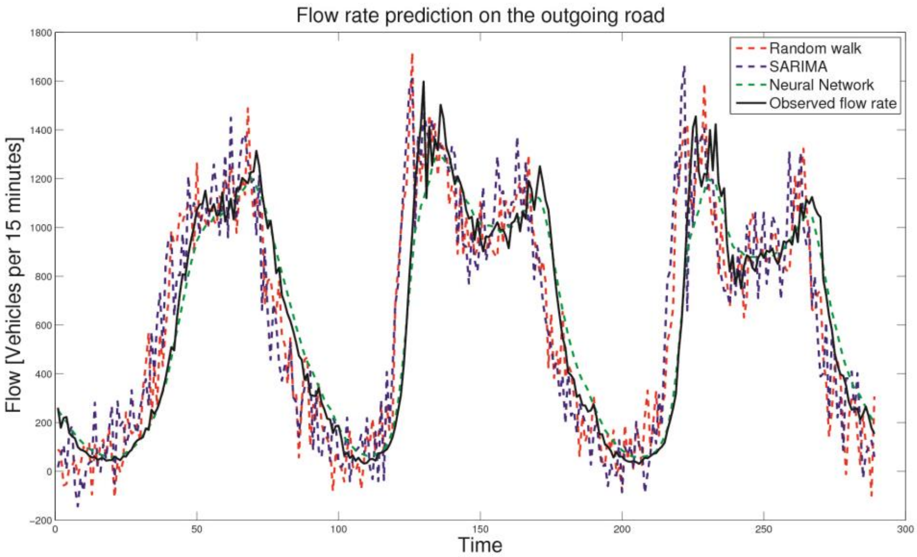


Figure 5: Model forecasts on the downstream road for 3 days

The table above indicates that the most accurate model is the neural network, however it is also the most computationally costly. Figure 5 shows the three forecast methods at the junction on the same graph and by visual inspection it is difficult to see which method is the most accurate at forecasting the traffic flow at the junction. However the graph does highlight the accuracy of all three methods, the forecasted data that uses the regression techniques is clear due to their stochastic nature, which is shown as frequent peaks and troughs. The neural network in comparison has a smoother forecast line.

The similarity in accuracy for the forecasting models is also reflected in the LWR graphs (Figures 5-7). The boundary conditions are the same for each hybrid model so any differences in the LWR graphs (between hybridizations) is only due to the forecast at the intersection; since there is little difference, the LWR density forecasts are roughly equal.

In single junction networks, such as the case study used in this paper, it is possible, and feasible to collect data throughout the day to inform the forecasts; therefore the simple regression models (random walk, SARIMA) used in the hybrid sense would be best given their speed and accuracy with their iterative updating. However the neural networks learning process allows for a much longer forecasting horizon (day, week, fortnight), therefore in cases where the data input is not constant the neural network would be the best option.

6. Conclusion

The paper for the first time compares hybrid macroscopic road traffic forecasting models, using the traffic flow and density data which it successfully models for a motorway junction in Hampshire, England. .

The hybrid algorithm performs successfully, accurately forecasting the density throughout the junction, showing the congestion and shockwaves developing and dissipating. Further development of the algorithm would involve a wider application to a broader motorway network with a more complex structure i.e. multiple intersections/junctions. As discussed in the previous section, the most suitable hybrid model to test with the LWR with the NN model component. Using real data captured at the exit of the downstream road, a thorough analysis of the bottleneck situation will be further analysed.

7. Acknowledgement

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